J. Inst. Eng. India Ser. C
DOI 10.1007/s40032-016-0299-9

REVIEW PAPER

An Intuitionistic Fuzzy Logic Models for Multicriteria Decision Making Under Uncertainty

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Received: 16 May 2012 / Accepted: 18 May 2016

Abstract The purpose of this paper is to enhance the applicability of the fuzzy sets for developing mathematical models for decision making under uncertainty, In general a decision making process consist of four stages, namely collection of information from various sources, compile the information, execute the information and finally take the decision/action. Only fuzzy sets theory is capable to quantifying the linguistic expression to mathematical form in complex situation. Intuitionistic fuzzy set (IFSs) which reflects the fact that the degree of non membership is not always equal to one minus degree of membership. There may be some degree of hesitation. Thus, there are some situations where IFS theory provides a more meaningful and applicable to cope with imprecise information present for solving multiple criteria decision making problem. This paper emphasis on IFSs, which is help for solving real world problem in uncertainty situation.

Keywords Intuitionistic fuzzy number · Multi criteria decision making · Fuzzy similarity

Introduction

Fuzzy sets are the sets with imprecise (vague) boundaries which was introduced by Zadeh [1]. It has attracted widespread attentions in various fields, especially where conventional mathematical techniques are less effectiveness, including social science, linguistic, psychology, economics and medical science [2, 3]. In such fields, variables are difficult to quantify and dependencies among variables are so ill-defined that precise characterization in terms of algebraic, differences or differential equations become almost impossible. Atanassov [4–7] dealt with IFSs which reflect the fact that the degree of non membership is not always equal to one minus degree of membership. Thus, there are some situations where IFSs theory provides a more meaningful and applicable to imprecise information present in real life applications.

The concept of IFSs can be viewed as an alternative approach to define a fuzzy set in case where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set [8, 9]. In general, the theory of IFSs is the generalization of fuzzy sets. Therefore, it is expected that IFSs could be used to simulate human decision-making processes and any activities necessary human expertise and knowledge. In the process of multiple criteria decision-making (MCDM) intuitionistic preference relation is a powerful tool to express the decision maker's intuitionistic preference information over the alternatives [10–12]. Burillo and Bustince [13] showed that the notion of vague sets coincides with that of IFS. Szmidt and Kacprzyk [14] proposed a non-probabilistic type of entropy measure for IFSs. De et al. [15] studied Sanchez's approach for medical diagnosis and extended this concept with the notion of IFSs theory. Turanli and Coker [16] introduced several types of fuzzy connectedness in intuitionistic fuzzy topological spaces. De et al. [15] defined some operations on IFSs. Szmidt and Kacprzyk [14, 17], Wang and Xin [18] discussed distance between IFSs. Bustince [13] presented different theorems for building intuitionistic fuzzy relations on a set with predetermined properties. Li and Cheng [19] studied similarity measures of IFSs and their application to

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pattern reorganizations. Szmidt and Kacprzyk [14] considered the use of IFSs for building MCDM models with uncertainty. However, it seems that so far there has been little research on MCDM in discrete decision situations and/or group decision-making using IFSs [20]. In this paper, MCDM using IFSs is investigated, in which attributes are explicitly considered, several corresponding linear programming models are constructed to general optimal weights of attributes, and the corresponding decision-making methods are also proposed.

This paper is organized as follows. The definitions and properties of IFSs are briefly introduced and describes MCDM models with intuitionistic fuzzy values in "Decision-Making Models Based on Fuzzy Data" section. "A Method to Estimate Weights of Alternatives" section describes an application of the proposed similarity measure to determine the alternative weights from intuitionistic preference relations. A numerical example and short conclusion are given in "Numerical Example" and "Conclusion" sections, respectively.

Decision-Making Models Based on Fuzzy Data

Fuzzy data was used for decision making; A fuzzy set is a class of objects with a continuum of membership grades. A membership function, which assigns to each object a grade of membership, is associated with each fuzzy set [21]. Usually, the membership grades range between 0 and 1. When the grade of membership for an object in a set is 1, this object is absolutely in that set; otherwise not in that set. It also projected as a framework for modelling uncertainty like vagueness, assign to each member of the universe of discourse a degree of membership between 0 and 1. Atanassov [4–7] introduced IFSs which reflects the fact that the degree of non membership is not always equal to one minus degree of membership. The IFSs theory has been studied and applied in different areas including decision-making. Now the modelling a decision problem, a situation may arise where the expert does not possess a clear idea of preferences between the alternatives [20]. However, in some situations the expert may provides his/her preferences for alternatives to a certain degree, but it is possible that he/she is not so sure about it. Since a uncertainty related to human cognitive processes through thinking, understanding, reorganization etc., present in experts' preferences values can be easily captured by intutionistic fuzzy values. Thus, in the process of decision-making intutionistic preferences relation is a powerful tool to express the decision maker's preferences information over the alternatives Szmidt and Kacprzyk [14].

Let $A_{IFS} = \{ <x_i,\ \mu_A(x_i),\ \upsilon_A(x_i) > \ : x_i \in X$ and $B_{IFS} = \{ <x_i,\ \mu_B(x_i),\ \upsilon_B(x_i) > \ : x_i \in X$ be the two IFS in $X = \{x_1,\ x_2,\ x_3,\ \ldots x_n\}$. Then the degree of similarity between the IFSs A_{IFS} and B_{IFS} may be evaluated with the help of the following model.

$$S_P(A_{IFS} - B_{IFS}) = 1 - \frac{1}{n}\sum_{i=1}^{n} \frac{\sqrt[p]{|\delta_A(x_i) - \delta_B(x_i)|^p + |\alpha_A(x_i) - \alpha_B(x_i)|^p}}{1 + \sqrt[p]{|\delta_A(x_i) - \delta_B(x_i)|^p + |\alpha_A(x_i) - \alpha_B(x_i)|^p}};\ 1 \le p < \infty \tag{1}$$

where, $\delta_A(x_i)$ and $\alpha_A(x_i)$ are defined in the following way

$$\delta_A(x_i) = \mu_A(x_i) + (1 - \mu_A(x_i) - \upsilon_A(x_i))\mu_A(x_i) \tag{2}$$

$$\alpha_A(x_i) = \upsilon_A(x_i) + (1 - \mu_A(x_i) - \upsilon_A(x_i))\upsilon_A(x_i) \tag{3}$$

In larger value of $S_P(A_{IFS}, B_{IFS})$ the more similarity between A_{IFS} and B_{IFS}. A comparison of the proposed similarity measure with the existing methods. The expressions of existing similarity measures are shown in Table 1.

Table 1 The expressions of existing similarity measures

The expressions of existing similarity measures	Examples	The proposed method
$S_p(A_{IFS}, B_{IFS}) = 1 - \frac{\sum_{i=1}^{n}\lvert S_A(x_i) - S_B(x_i)\rvert}{2n}$ where, $S_A(x_i) = \mu_A(x_i) - \nu_A(x_i)$ and $S_B(x_i) = \mu_B(x_i) - \nu_B(x_i)$	Example 1 : consider $A_{IFS} = \{\langle x, 0.4, 0\rangle\}$ and $B_{IFS} = \{\langle x, 0.6, 0.2\rangle\}$ $S^C = (A_{IFS}, B_{IFS}) = 1$	$P = 1$, we have $S_p = (A_{IFS}, B_{IFS}) = 0.76$
$S^z(A_{IFS}, B_{IFS}) = 1 - \frac{1}{2n}\sum_{i=1}^{n}(\lvert\delta_A(x_i) - \delta_B(x_i)\rvert + \lvert\alpha_A(x_i) - \alpha_B(x_i)\rvert)$	Example 2 : consider $A_{IFS} = \{\langle x, 0, 0\rangle \mid x \in X\}$ $B^1_{IFS} = \{\langle x, 0.2, 0.8\rangle \mid x \in X\}$ $B^2_{IFS} = \{\langle x, 0.4, 0.6\rangle \mid x \in X\}$ $B^3_{IFS} = \{\langle x, 0.3, 0.7\rangle \mid x \in X\}$ $S^z(A_{IFS}, B^1_{IFS}) = S^z(A_{IFS}, B^2_{IFS}) = S^z(A_{IFS}, B^3_{IFS}) = 0.500$	$P = 1$, we have $S_p(A_{IFS}, B^1_{IFS}) = 0.548$ $S_p(A_{IFS}, B^2_{IFS}) = 0.581$ $S_p(A_{IFS}, B^3_{IFS}) = 0.568$

A Method to Estimate Weights of Alternatives

In order to determine the weights of alternatives in a decision-making process, a methodology has been developed in this section, based on intuitionistic preference relation, which can be described as follows:

Step 1 For a MCDM problem, let 'n' numbers of alternative A_1, $A_{1,\ \ldots}$, A_n, are set by the expert where the weights are unknown. The expert provides his/her intuitionistic preference for every pair of alternatives (A_i, A_j) and constructs intuitionistic preference relations as follows:

$R_{IPR} = (r_{ij})_{n\times n}$ where $r_{ij} = (\mu_{ij}, \upsilon_{ij})$, $0 \leq \mu_{ij} + \upsilon_{ij} \leq 1$ for all i, j, = 1, 2, … n. From Eq. 1, we may write $\mu_{ij} - \upsilon_{ij}$, $\upsilon_{ij} - \mu_{ij}$, μ_{ij} - $\upsilon_{ij} - 0.5$ for all i, j = 1, 2, … n. Therefore, for 'n' number of alternatives the intuitionistic preference relations

$R_{IPR} - (r_{ij})_{n\times n}$, given by the expert, may be expressed as follows:

$$R_{IPR} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_j \\ \vdots \\ A_n \end{array}\begin{array}{c} \begin{array}{ccccccc} A_1 & A_2 & \cdots & A_j & \cdots & A_n \end{array} \\ \begin{pmatrix} (0.5,\ 0.5) & (\mu_{12},\ \nu_{12}) & \cdots & (\mu_{1j},\ \nu_{1j}) & \cdots & (\mu_{1n},\ \nu_{1n}) \\ (\mu_{21},\ \nu_{21}) & (0.5,\ 0.5) & \cdots & (\mu_{2j},\ \nu_{2j}) & \cdots & (\mu_{2n},\ \nu_{2n}) \\ \vdots & \vdots & \cdots & \vdots & & \vdots \\ (\mu_{i1},\ \nu_{i1}) & (\mu_{i2},\ \nu_{i2}) & \cdots & (\mu_{ij},\ \nu_{ij}) & \cdots & (\mu_{in},\ \nu_{in}) \\ \vdots & \vdots & \cdots & \vdots & & \vdots \\ (\mu_{n1},\ \nu_{n1}) & (\mu_{n2},\ \nu_{n2}) & \cdots & (\mu_{nj},\ \nu_{nj}) & \cdots & (0.5,\ 0.5) \end{pmatrix}\end{array}$$

Step 2 Among the 'n' alternatives let us consider the pair (A_i, A_j). If alternative A_i is definitely preferred over A_j then the corresponding intuitionistic preference relation is $(1, 0)$; where the certainty degree to which A_i is preferred to A_j is 1, and the certainty degree to which A_i is non-preferred to A_j is 0.

Step 3 If for the pair of alternative (A_i, A_j) the intuitionistic preference value is $r_{ij} - (\mu_{ij}, \nu_{ij})$, then calculate the similarity between r_{ij} and $(1, 0)$ using (1). In this way, for all the pair of alternatives (A_i, A_j) ∀ i, j = 1, 2, … n, i ≠ j, "c_2" similarly values can be calculated and subsequently it has been expressed in a concise manner in the form of a matrix as follows:

$$R' = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_j \\ \vdots \\ A_n \end{array}\begin{array}{c} \begin{array}{cccccc} A_1 & A_2 & \cdots & A_j & \cdots & A_n \end{array} \\ \begin{pmatrix} S_{p11} & S_{p12} & \cdots & S_{p1j} & \cdots & S_{p1n} \\ S_{p21} & S_{p22} & \cdots & S_{p2j} & \cdots & S_{p2n} \\ \vdots & \vdots & \cdots & \vdots & & \vdots \\ S_{pi1} & S_{pi2} & \cdots & S_{pij} & \cdots & S_{pin} \\ \vdots & \vdots & \cdots & \vdots & & \vdots \\ S_{pn1} & S_{pn2} & \cdots & S_{pnj} & \cdots & S_{pnn} \end{pmatrix} \\ \cdots \end{array}$$

where $S_{pij} - S_p(r_{ij}, r_j); 1 \leq p < \infty$; assuming $r-(1, 0)$.

Step 4 Finally, the weight of the alternative A_i is defined by

$$W_p(A_i) - 1 \underset{i \neq j}{\overset{min}{\leq j \leq}} n\{S_{pij}\} \quad \text{for} 1 \quad \leq p \ < \infty$$

Thus, the weight vectors of the alternatives $A_1, A_2, \ldots, A_n$ are $W_p(A_1), W_p(A_2), \ldots, W_p(A_n)$, respectively, for $1 \leq p < \infty$.

Numerical Example

A decision maker intends to buy an air condition system. He/She has five alternatives to choose, namely $A = \{A_1, A_2, A_3, A_4, A_5\}$.. Taking into consideration various factors, like price, quality, service, user-friendliness the

$$R_{IPR} = \left(r_{ij}\right)_{5x5} \quad \begin{matrix} & A_1 & A_2 & A_3 & A_4 & A_5 \\ A_1 & (0.5,\ 0.5) & (0.6,\ 0.3) & (0.4,\ 0.2) & (0.7,\ 0.2) & (0.4,\ 0.5) \\ A_2 & (0.5,\ 0.5) & (0.5,\ 0.5) & (0.5,\ 0.5) & (0.5,\ 0.5) & (0.5,\ 0.5) \\ A_3 & (0.5,\ 0.5) & (0.5,\ 0.5) & (0.5,\ 0.5) & (0.5,\ 0.5) & (0.5,\ 0.5) \\ A_4 & (0.5,\ 0.5) & (0.5,\ 0.5) & (0.5,\ 0.5) & (0.5,\ 0.5) & (0.5,\ 0.5) \\ A_5 & (0.5,\ 0.5) & (0.5,\ 0.5) & (0.5,\ 0.5) & (0.5,\ 0.5) & (0.5,\ 0.5) \end{matrix}$$

decision maker constructs the intuitionistic preferences relation as follows

In R_{IPR}, the element $r_{12} = (0.6, 0.3)$ is composed by the certainty degree 0.6 to which A_1 is preferred to A_2 and the certainty degree 0.3 to which A_1 is non-preferred to A_2. The other elements in R_{IPR} may be interpreted in the same way.

Utilizing (1) the similarity measure $(for\, p{-}1)$ between each entry of R_{IPR} and $(1, 0)$ is computed and the matrix R can be constructed as follows:

$$R_{IPR} = \left(r_{ij}\right)_{5x5} \quad \begin{matrix} & A_1 & A_2 & A_3 & A_4 & A_5 \\ A_1 & (0.5,\ 0.5) & (0.6,\ 0.3) & (0.4,\ 0.2) & (0.7,\ 0.2) & (0.4,\ 0.5) \\ A_2 & (0.5,\ 0.5) & (0.5,\ 0.5) & (0.5,\ 0.5) & (0.5,\ 0.5) & (0.5,\ 0.5) \\ A_3 & (0.5,\ 0.5) & (0.5,\ 0.5) & (0.5,\ 0.5) & (0.5,\ 0.5) & (0.5,\ 0.5) \\ A_4 & (0.5,\ 0.5) & (0.5,\ 0.5) & (0.5,\ 0.5) & (0.5,\ 0.5) & (0.5,\ 0.5) \\ A_5 & (0.5,\ 0.5) & (0.5,\ 0.5) & (0.5,\ 0.5) & (0.5,\ 0.5) & (0.5,\ 0.5) \end{matrix}$$

Finally, utilizing (6.9) the weights of the alternatives are obtained as follows:

$$W(A_1) - 0.473, W(A_2) - 0.429, W(A_3) - 0.438, W(A_4) - 0.377 \text{ and } W(A_5) - 0.529$$

Hence, the best air-condition system is A_5, since it has the highest priority weight.

Note: R_{IPR}: Intuitionistic preference relation between two alternatives.

Conclusion

Decision-making is a crucial topic in fuzzy systems, which we have studied from different points of view. For this purpose, a new similarity measure of IFSs has been proposed. Furthermore, a numerical example has also been provided to make a comparison between the proposed similarity measure and the exciting models. Finally, an air conditioning system selection problem has been presented to illustrate the application of the proposed measure. This allows us to use flexible ways to simulate real MCDM under uncertainty, thereby building more realistic scenarios describing possible future events. In conclusion, this proposed model may be applicable for decision-making problem in various fields.

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